

Exercice 1: 5 points

1.1. $e^x = 1 \Leftrightarrow e^x = e^0 \Leftrightarrow x = 0$ 1 pt

1.2. $e^x > 0 \forall x \in \mathbb{R} \Rightarrow e^{2x} + 1 > 0 \forall x \in \mathbb{R}$
 $\Rightarrow e^x + 1 = 0$ n'a pas de solution 1 pt

1.3. $e^x < \frac{1}{e^{2x+1}} \Leftrightarrow e^x e^{2x+1} < 1$ (car $e^{2x+1} > 0$) 0,5 pt

$\Leftrightarrow e^{3x+1} < e^0 \Leftrightarrow 3x+1 < 0 \Leftrightarrow x < -\frac{1}{3}$ 1,5 pt

1.4. du signe de $e^{3x} - 2e^x + 1$ (car $e^x > 0 \forall x \in \mathbb{R}$)
 $X = e^x$ (1) $\Leftrightarrow X^3 - 2X + 1 = (X-1)^2 \geq 0 \forall X$
 $\Rightarrow (1) \geq 0 \forall x \Rightarrow \Gamma = \mathbb{R}$ 1,5 pt

Exercice 2: 5 points

$f(x) = x^5 + 2x^4 + 2x^3 + x + 1$

$f'(x) = 5x^4 + 4x^3 + 3x^2 + 1$

$f''(x) = 20x^3 + 12x^2 + 6x = 2x(10x^2 + 6x + 3)$

$\Delta = 36 - 4 \cdot 10 \cdot 3 < 0 \Rightarrow 10x^2 + 6x + 3 > 0 \forall x \in \mathbb{R}$

$\Rightarrow f'(x)$ du signe de x

x	$-\infty$	0	$+\infty$
		$-$	$+$
$f'(x)$		$-$	$+$
$f'(0)$		$\rightarrow 1$	
$f'(x)$		$+$	
f			\rightarrow

$f'(0) = 1$
 $\Rightarrow \exists$ min de f'
 $\Rightarrow f'(x) > 0 \forall x$

Ex 3: 5 points

$f(x) = \frac{e^x}{1+x}$ ~~def~~ définie et dérivable sur $\mathbb{R} \setminus \{-1\}$

$u = e^x \quad u' = e^x \quad v = 1+x \quad v' = 1$

$f'(x) = \frac{e^x(1+x) - e^x}{(1+x)^2} = x \frac{e^x}{(1+x)^2}$

$e^x > 0 \quad \forall x \in \mathbb{R}$ et $(1+x)^2 \geq 0 \quad \forall x \in \mathbb{R} \Rightarrow f'$ des signes

x	$-\infty$	-1	0	$+\infty$
$(1+x)^2$	+	ϕ	+	+
x	-		-	+
$f'(x)$	-		-	+
f			1	

1 min local

Ex 4: 5 points

```

i) def exponentielle (n):
    fact = 1
    e = 0
    for i in range (n+1):
        if i > 1:
            fact = fact * i
        e = e + (1 / fact)
    return e
    
```

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iii) def factorielle 2 (n):
    if n <= 1:
        return 1
    else:
        return n * factorielle (n-1)
    
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def exponentielle 3 (n):
    return sum ([ 1 / factorielle 2 (i) for i in range (n+1) ])
    
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ii) def factorielle (n):
    if n <= 1:
        return 1
    else:
        fact = 1
        for i in range (2, n+1):
            fact = fact * i
        return fact
    def exponentielle (n):
        e = 0
        for i in range (n+1):
            e = e + 1 / factorielle (i)
        return e
    
```